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FRICTIONAL HEATING DURING SLIDING OF TWO SEMI-SPACES WITH ARBITRARY THERMAL NONLINEARITY

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Abstract: Analytical and numerical solution for transient thermal problems of friction were presented for semi limited bodies made from thermosensitive materials in which coefficient of thermal conductivity and specific heat arbitrarily depend on the temperature (materials with arbitrary non-linearity). With the constant power of friction assumption and imperfect thermal contact linearization of nonlinear problems formulated initial-boundary thermal conductivity, using Kirchhoff transformation is partial. In order to complete linearization, method of successive approximations was used. On the basis of obtained solutions a numerical analysis of two friction systems in which one element is constant (cermet FMC-845) and another is variable (grey iron ChNMKh or aluminum-based composite alloy AL MMC) was conducted.

Key words: Frictional Heating, Arbitrary Thermal Non-Linearity, Thermosensitive Materials

1. INTRODUCTION

Developing analytical or analytical – numerical solutions of heat conduction problems with regard to the frictional heating is necessary in designing new types of friction nodes (eg. the choice of friction material, estimation of the temperature level, selection of the process operational parameters, etc.). In such modeling method the real friction pairs are replaced by systems such as: strip - half-space, strip - strip or semi-space - semi-space.

Friction heat problem solution for a homogeneous semi-space sliding at constant speed on the surface of the second semi-space, with constant thermophysical materials properties and perfect thermal contact condition fulfillment are presented in Grylitsky (1996) monograph, and solution taking into account imperfect thermal contact – in work: Sazonov (2008).

Frictional heating during two semi-space, strip and semi-space, slipping with a constant delay and the two strips system with regard to various kinds of conditions of thermal contact was examined in works: Nosko et al. (2009), Yevtushenko et al. (2013, 2014), Barber (1976). Details on methods for solving one-dimensional thermal problems of friction with constant thermophysical materials properties are given in monograph Kuciej (2012).

The analysis of literature sources concerning analytical or analytical-numerical modeling of heat generation processes due to frictional forces, shows that models considering constant thermophysical properties of friction materials are very well developed. Whereas, solutions involving a non-linearity of frictional materials pairs have been so far developed in an insufficient number of cases.

Homogeneous semi-space surface frictional heating with linear dependence of thermal conductivity coefficients and specific heat of the temperature at a constant ratio of the thermal diffusivity (material with a simple non-linearity) was examined in the work: Abdel-Aal, (1997), Abdel-Aal et al. (1997), Abdel-Aal and Smith (1998, 1998b). Methods of solving one-dimensional initial-boundary problems of thermal conductivity for the two semi-spaces sliding against one another at a constant linear velocity or semi-space made of materials with a simple non-linearity, are proposed in the work: Och (2013), Evtushenko et al. (2014), Yevtushenko et al. (2014, 2014b), whereas solutions involving arbitrary non-linearity (thermophysical properties of materials change under the influence of temperature in any way) in the work: Yevtushenko et al. (2014c, d).

Surveys of analytical and numerical methods for solving initial-boundary problems of heat conduction for materials with temperature-dependent thermal properties are presented in the work: Kushnir and Popovych (2011), Yevtushenko and Kuciej (2012).

The present work is a continuation of studies presented in article Och (2013), where case of simple thermal nonlinearity of materials was considered. Whereas in this work friction elements materials are characterized by arbitrary non-linearity.

2. STATEMENT OF THE PROBLEM

Let the two semi-limited thermally sensitive bodies with the same initial temperature T_0 be compressed at infinite and constant pressure p_0 in the parallel direction to the z axis of the Cartesian coordinate system $Oxyz$ (Fig. 1). At initial point of time $t = 0$ bodies start to slip with a constant speed V_0 in the positive direction of y-axis. Due to the friction forces, on the contact surface $z = 0$ heat is generated, which penetrates into the contacting bodies - heating them.

It was assumed, that the sum of heat fluxes intensities directed perpendicularly from the contact surfaces to the inside of each contacting body, is equal to the specific power of friction forces $q_0 = fV_0p_0$ (Yevtushenko and Kuciej, 2012). Whereas the heat transfer through the surface of the friction takes place with constant coefficient of contact thermal conductivity h (Podstrigach, 1963; Barber, 1970).

Further in these article, all values referring to the upper and lower semi-spaces will respectively have subscripts 1 and 2.

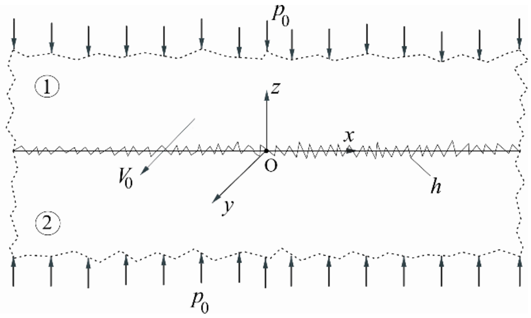

Fig. 1. The scheme of frictional heating

Both, coefficient of thermal conductivity K_l and specific heat c_l, $l = 1, 2$ of the two semi-spaces materials depend on temperature T:

$$K_l(T) = K_{l,0}\, K_l^*(T),\ \ K_{l,0} \equiv K_l(T_0)$$
$$c_l(T) = c_{l,0}\, c_l^*(T), c_{l,0} \equiv c_l(T_0) \tag{1}$$

$K_l^*(T), c_l^*(T)$ – dimensionless functions. Densities ρ_l, $l = 1, 2$ of considered bodies materials are constant.

Taking into account mentioned above assumptions, the distribution of transient temperature field $T(z,t)$ in semi-spaces is found from the following heat conduction problem boundary-value solution:

$$\frac{\partial^2 T_1^*}{\partial \zeta^2} = \frac{1}{k_0^* k_1^*(T_1^*)}\frac{\partial T_1^*}{\partial \tau},\ \ \zeta > 0,\ \tau > 0 \tag{2}$$

$$\frac{\partial^2 T_2^*}{\partial \zeta^2} = \frac{1}{k_2^*(T_2^*)}\frac{\partial T_2^*}{\partial \tau},\ \ \zeta < 0,\ \tau > 0 \tag{3}$$

$$K_2^*(T_2^*)\frac{\partial T_2^*}{\partial \zeta}\bigg|_{\zeta=0^-} - K_0^*\ K_1^*(T_1^*)\frac{\partial T_1^*}{\partial \zeta}\bigg|_{\zeta=0^+} = 1,\ \ \tau > 0 \tag{4}$$

$$K_2^*(T_2^*)\frac{\partial T_2^*}{\partial \zeta}\bigg|_{\zeta=0^-} + K_0^*\ K_1^*(T_1^*)\frac{\partial T_1^*}{\partial \zeta}\bigg|_{\zeta=0^+} = Bi\,[T_1^*(0^+,\tau) - T_2^*(0^-,\tau)],\ \ \tau > 0 \tag{5}$$

$$T_l^*(\zeta,\tau) \to T_0^*,\ \ |\zeta| \to \infty,\ \ l = 1{,}2 \tag{6}$$

$$T_l(\zeta,0) = T_0^*,\ \ |\zeta| < \infty,\ \ l = 1{,}2 \tag{7}$$

where:

$$\zeta = \frac{z}{a},\ \ \tau = \frac{k_2 t}{a^2},\ \ K_0^* = \frac{K_{1,0}}{K_{2,0}},\ \ k_0^* = K_0^*\frac{\rho_2 c_{2,0}}{\rho_1 c_{1,0}},\ \ Bi = \frac{h\,a}{K_{2,0}} \tag{8}$$

$$T_a = \frac{q_0 a}{K_{2,0}},\ \ T_l^* = \frac{T_l}{T_a},\ T_0^* = \frac{T_0}{T_a}, l = 1{,}2 \tag{9}$$

$$k_l^*(T_l^*) = \frac{K_l^*(T_l^*)}{c_l^*(T_l^*)}, l = 1{,}2 \tag{10}$$

a – is a characteristic linear dimension. Further, as the value of this parameter, we take the effective depth of heat penetration, i.e. the distance from the contact surface, on which the temperature is equal to 5% of the maximal temperature on the surface of friction (Chichinadze et al., 1979).

3. LINEARIZATION OF THE PROBLEM

In order to linearize the non-linear boundary-value heat conduction problem of friction (2) – (7) we introduce the Kirchhoff's function (Kirchhoff, 1894):

$$\Theta_l(\zeta,\tau) = \int_{T_0^*}^{T^*} K_l^*(T_l^*)\, dT_l^*,\ \ l = 1{,}2 \tag{11}$$

As a result, we obtained a sequence of linear boundary-value problems in relation to the functions $\Theta_l^{(i)}(\zeta,\tau)$, $i = 0{,}1,\ldots$, that are successive approximations of the sought Kirchhoff's function (Yevtushenko et al., 2014c, d):

$$\frac{\partial^2 \Theta_1^{(i)}}{\partial \zeta^2} = \frac{1}{k_0^*\, k_1^{*(i)}}\frac{\partial \Theta_1^{(i)}}{\partial \tau}, \zeta > 0,\ \tau > 0 \tag{12}$$

$$\frac{\partial^2 \Theta_2^{(i)}}{\partial \zeta^2} = \frac{1}{k_2^{*(i)}}\frac{\partial \Theta_2^{(i)}}{\partial \tau}, \zeta < 0,\ \tau > 0 \tag{13}$$

$$\frac{\partial \Theta_2^{(i)}}{\partial \zeta}\bigg|_{\zeta=0} - K_0^*\frac{\partial \Theta_1^{(i)}}{\partial \zeta}\bigg|_{\zeta=0} = 1, \tau > 0 \tag{14}$$

$$\frac{\partial \Theta_2^{(i)}}{\partial \zeta}\bigg|_{\zeta=0} + K_0^*\frac{\partial \Theta_1^{(i)}}{\partial \zeta}\bigg|_{\zeta=0} = Bi^{(i)}\,[\Theta_1^{(i)}(0,\tau) - \Theta_2^{(i)}(0,\tau)], \tau > 0 \tag{15}$$

$$\Theta_l^{(i)}(\zeta,\tau) \to 0,\ \ |\zeta| \to \infty,\ \ \tau \ge 0,\ \ l = 1{,}2 \tag{16}$$

$$\Theta_l^{(i)}(\zeta,0) = 0,\ \ |\zeta| < \infty,\ \ l = 1{,}2 \tag{17}$$

gdzie:

$$k_l^{*(0)} = 1, k_l^{*(i)} = k_l^*\left\{T^*\left[\Theta_l^{(i-1)}(\zeta^*,\tau^*)\right]\right\}$$
$$i = 1{,}2,\ldots;\ \ l = 1{,}2 \tag{18}$$

$$Bi^{(0)} = Bi,$$
$$Bi^{(i)} = Bi\frac{\{T^*[\Theta_1^{(i-1)}(0,\tau^*)] - T^*[\Theta_2^{(i-1)}(0,\tau^*)]\}}{[\Theta_1^{(i-1)}(0,\tau^*) - \Theta_2^{(i-1)}(0,\tau^*)]},\ \ i = 1{,}2,\ldots, \tag{19}$$

(ζ^*,τ^*) – these are established values of dimensionless coordinate and time (8), for which we perform iteration.

4. KIRCHHOFF FUNCTION

By applying the Laplace integral transform (Sneddon, 1972):

$$\bar{\Theta}_l^{(i)}(\zeta,p) \equiv L\,[\Theta_l^{(i)}(\zeta,\tau); p] = \int_0^\infty \Theta_l^{(i)}(\zeta,\tau)\, e^{-p\tau} d\tau \tag{20}$$

to the linear boundary-value problem (11)–(19), we obtain the following boundary problem for two ordinary differential equations of the second order:

$$\frac{d^2\bar{\Theta}_1^{(i)}(\zeta,p)}{d\zeta^2} - \frac{p}{k_0^* k_1^{*(i)}}\bar{\Theta}_1^{(i)}(\zeta,p) = 0,\ \ \zeta > 0 \tag{21}$$

$$\frac{d^2\bar{\Theta}_2^{(i)}(\zeta,p)}{d\zeta^2} - \frac{p}{k_2^{*(i)}}\bar{\Theta}_2^{(i)}(\zeta,p) = 0,\ \ \zeta < 0 \tag{22}$$

$$\frac{d\bar{\Theta}_2^{(i)}}{d\zeta}\bigg|_{\zeta=0} - K_0^*\frac{d\bar{\Theta}_1^{(i)}}{d\zeta}\bigg|_{\zeta=0} = \frac{1}{p} \tag{23}$$

$$\frac{d\bar{\Theta}_2^{(i)}}{d\zeta}\bigg|_{\zeta=0} + K_0^*\frac{d\bar{\Theta}_1^{(i)}}{d\zeta}\bigg|_{\zeta=0} = Bi^{(i)}[\bar{\Theta}_1^{(i)}(0,p) - \bar{\Theta}_2^{(i)}(0,p)] \tag{24}$$

$$\bar{\Theta}_l^{(i)}(\zeta,p) \to 0,\ \ |\zeta| \to \infty,\ \ l = 1{,}2 \tag{25}$$

Solution to the problem (21)-(25) takes form:

$$\bar{\Theta}_1^{(i)}(\zeta,p) = \frac{\sqrt{k^{*(i)}}e^{-\zeta_1^{(i)}\sqrt{p}}}{2\varepsilon p(\sqrt{p}+\beta^{(i)})}\left(1 + Bi^{(i)}\sqrt{\frac{k_2^{*(i)}}{p}}\right),\ \ \zeta \ge 0,$$
$$\bar{\Theta}_2^{(i)}(\zeta,p) = \frac{e^{-\zeta_2^{(i)}\sqrt{p}}}{2\varepsilon p(\sqrt{p}+\beta^{(i)})}\left(\varepsilon + Bi^{(i)}\sqrt{\frac{k_1^{*(i)}}{p}}\right),\ \ \zeta \le 0, \tag{26}$$

$$\zeta_1^{(i)} = \frac{\zeta}{\sqrt{k_0^* k_1^{*(i)}}}, \zeta_2^{(i)} = \frac{|\zeta|}{\sqrt{k_2^{*(i)}}}, \quad (27)$$

$$\beta^{(i)} = 0.5\varepsilon^{-1} Bi^{(i)} m^{(i)}, \quad m^{(i)} = \sqrt{k^{*(i)}} + \varepsilon, \quad (28)$$

$$k^{*(i)} = \frac{k_1^{*(i)}}{k_2^{*(i)}}, \varepsilon = \frac{K_0^*}{\sqrt{k_0^*}}. \quad (29)$$

Applying the inversion formulae (Bateman and Erdelyi, 1954):

$$L^{-1}\left[\frac{\beta\, e^{-\alpha\sqrt{p}}}{p\sqrt{p}(\sqrt{p}+\beta)}; \tau\right] = \Phi(\alpha,\tau) - \beta^{-1}\Psi(\alpha,\beta,\tau) \quad (30)$$

$$L^{-1}\left[\frac{\beta\, e^{-\alpha\sqrt{p}}}{p(\sqrt{p}+\beta)}; \tau\right] = \Psi(\alpha,\beta,\tau), \alpha,\beta \ge 0 \quad (31)$$

$$\Phi(\alpha,\tau) = 2\sqrt{\tau}\,\mathrm{ierfc}\left(\frac{\alpha}{2\sqrt{\tau}}\right), \tau \ge 0 \quad (32)$$

$$\Psi(\alpha,\beta,\tau) = \mathrm{erfc}\left(\frac{\alpha}{2\sqrt{\tau}}\right) - e^{\alpha\beta+\beta^2\tau}\mathrm{erfc}\left(\frac{\alpha}{2\sqrt{\tau}} + \beta\sqrt{\tau}\right) \quad (33)$$

to the solutions (26)-(29), we obtain:

$$\Theta_1^{(i)}(\zeta,\tau) = \frac{1}{m^{(i)}}\left[\Phi\left(\zeta_1^{(i)},\tau\right) + \gamma_1^{(i)}\Psi\left(\zeta_1^{(i)},\beta^{(i)},\tau\right)\right], \quad \zeta \ge 0, \tau \ge 0 \quad (34)$$

$$\Theta_2^{(i)}(\zeta,\tau) = \frac{\sqrt{k^{*(i)}}}{m^{(i)}}\left[\Phi(\zeta_2^{(i)},\tau) + \gamma_2^{(i)}\Psi(\zeta_2^{(i)},\beta^{(i)},\tau)\right] \quad \zeta \le 0,\ \tau \ge 0 \quad (35)$$

$$\gamma_1^{(i)} = \frac{(\sqrt{k^{*(i)}}-\varepsilon)}{\sqrt{k_2^{*(i)}}(\sqrt{k^{*(i)}}+\varepsilon)\, Bi^{(i)}}, \gamma_2^{(i)} = \frac{\varepsilon(\varepsilon-\sqrt{k^{*(i)}})}{\sqrt{k_1^{*(i)}}(\sqrt{k^{*(i)}}+\varepsilon)\, Bi^{(i)}} \quad (36)$$

In particular case $\zeta = 0$ from solutions (34)–(36) we obtain:

$$\Theta_1^{(i)}(0,\tau) = \frac{1}{m^{(i)}}\left[\varphi(\tau) + \gamma_1^{(i)}\psi(\beta^{(i)},\tau)\right], \tau \ge 0, \quad (37)$$

$$\Theta_2^{(i)}(0,\tau) = \frac{\sqrt{k^{*(i)}}}{m^{(i)}}\left[\varphi(\tau) + \gamma_1^{(i)}\psi(\beta^{(i)},\tau)\right], \quad (38)$$

$$\varphi(\tau) = 2\sqrt{\frac{\tau}{\pi}}, \quad (39)$$

$$\psi(\beta^{(i)},\tau) = 1 - e^{\beta^{(i)2}\tau}\mathrm{erfc}(\beta^{(i)}\sqrt{\tau}). \quad (40)$$

5. ITERATIVE SCHEME

To find the relation between temperature and Kirchhoff's function the form of functions $K_l^*(T_l^*)$ and $c_l^*(T_l^*)$ should be specified (1). We assume that these are polynomials:

$$K_l^*(T_l^*) = \sum_{n=0}^{N_l} a_{l,n}(T_l^*)^n, \; c_l^*(T^*) = \sum_{n=0}^{M_l} b_{l,n}(T_l^*)^n \quad (41)$$

where $a_{l,n}$ and $b_{l,n}$ - are known coefficients for materials of each considered friction pairs. We use both equations (41) to find the function $k_l^*(T_l^*)$ (10). In addition, substituting the function $K_l^*(T_l^*)$ (41) into equation (10), after integration we obtain the relation between temperature and Kirchhoff's function:

$$\Theta_l(\zeta^*,\tau^*) = \sum_{n=0}^{n} a'_{l,n}\{[T^*(\zeta^*,\tau^*)]^{n+1} - (T_0^*)^{n+1}\} \quad (42)$$

where $a'_{l,n} = a_{l,n}/(1+n)$, $l = 1, 2$. Using the method of least squares we located dependency inverse to (42):

$$T_l^*(\zeta^*,\tau^*) = \sum_{n=0}^{N_l} c_{l,n}[\Theta_l(\zeta^*,\tau^*)]^n, l = 1, 2 \quad (43)$$

where$c_{l,n}$ - are known coefficients.

For the zero approximation ($i = 0,\ k_l^{(0)} = 1,\ Bi^{(0)} = Bi$) solutions (34) – (40) are coincident with boundary-value problems solutions with constant thermo-physical properties of materials (Sazonov, 2008). The relation between temperature and Kirchhoff's function at this iterative step is linear:

$$T_l^{*(0)}(\zeta^*,\tau^*) = \Theta_l^{(0)}(\zeta^*,\tau^*) + T_0^*,\ l = 1,2. \quad (44)$$

For successive approximations ($i \ge 1$) we begin with calculating the values of $k_l^{*(i)},\ l = 1,2$, (17) and $Bi^{(i)}$ (18) and finding the function $\Theta_l^{(i)}(\zeta^*,\tau^*)$ (34) - (36) or $\Theta_l^{(i)}(0,\tau^*)$ (37) – (40), $l = 1,2$. Then from the formula (43), we obtain an approximation of the dimensionless temperature $T_l^{*(i)}(\zeta^*,\tau^*)$ or $T_l^{*(i)}(0,\tau^*)$, $l = 1,2$. The convergence of an iterative process in (ζ^*,τ^*) is monitored by checking the inequality (Euclidean norm):

$$\sqrt{\left(\frac{T_1^{*(i)} - T_2^{*(i-1)}}{T_1^{*(i)}}\right)^2 + \left(\frac{T_2^{*(i)} - T_2^{*(i-1)}}{T_2^{*(i)}}\right)^2} \le 10^{-6}. \quad (45)$$

6. NUMERICAL ANALYSIS AND CONCLUSIONS

The calculations have been made for the following friction pairs: gray iron ChNMKh - cermet FMC-845 and composite on aluminum alloy base AL MMC - FMC-845. These materials are used to produce friction elements of braking systems (Chichinadze et al., 1986; Kim el al., 2008). The values of the thermophysical properties of selected materials at temperature $T_0 = 20\,^\circ\mathrm{C}$ are given in Tab. 1. Graphs of functions $K_l^*(T^*)$ and $c_l^*(T^*)$ (41), for the consideraded friction materials are shown in Fig. 2, and the corresponding values of the coefficients $a_{l,n}$, $b_{l,n}$ and $c_{l,n}$, $n = 0,1,2,3$ are given in Tab. 2. We can notice that the coefficient of thermal conductivity of gray iron ChNMKh and cermet's FMC-845 decreases with increasing temperature, and AL MMC increases in the range $0 \le T^* \le 0.5$, and then for $0.5 < T^* \le 1.3$ decreases, and next for $T^* > 1.3$ rise again. Specific heat of all considered materials increases with increase of temperature. Calculations were performed for the following input parameters: $q_0 = 1\mathrm{MW/m}$, $a = 5$ mm, $T_a = 204\,^\circ\mathrm{C}$ and $Bi = 5$.

Tab. 1. The thermo-physical coefficients values of the materials at the$T_0 = 20\,^\circ\mathrm{C}$

Materials	K_0 W/(m · C)	c_0 J/(kg · C)	ρ kg/m^3
FMC-845 (Chichinadze et al., 1986)	24.5	392.2	6000
ChNMKh (Chichinadze et al., 1986)	51	500.1	7100
AL MMC (Kim el al., 2008)	155.75	874	2730

Evolutions of the dimensionless temperature on the contact surface of two selected friction pairs ChNMKh–FMC-845 and AL MMC–FMC-845 are presented in Fig. 3. The calculations have been made with taking into account friction materials thermal properties changes influenced by temperature (solid lines) and at the constant thermal properties of these materials (dashed lines).

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Tab. 2. The approximation coefficients values

	n	0	1	2	3
FMC-845	a_n	1	0.001799	-0.019018	0.001953
	b_n	1	0.000547	0.048323	0.001003
	c_n	0.098041	0.99977	0.001746	0.005215
ChNMKh	a_n	1	-0.112		
	b_n	1	0.325214	0.065993	-0.027336
	c_n	0.098543	1.012837	0.05264	0.011315
AL MMC	a_n	1	0.356782	-0.421682	0.141147
	b_n	1	0.209544	-0.147707	0.062199
	c_n	0.097949	0.926329	0.002449	-0.00639

The temperature increases monotonically during the whole process of frictional heating, which is characteristic for the heating process at the constant power of friction forces (Grylitsky, 1996). The effect of thermal sensitivity with temperature increase is most noticeable for the cermets in the case of frictional pair ChNMKh-FMC-845 (Fig. 3a). However for gray iron ChNMKh temperature difference after certain time from beginning of the frictional heating process ($\tau \approx 0.5$), reaches predetermined value. Taking into account FMC-845 thermal sensibility results in temperature increase, and decrease in ChNMKh, compared to the temperature values found at constant thermophysical properties of materials. This is caused by cermet FMC-845 coefficient of thermal conductivity (1%) and specific heat (2.5%) decrease at $0 \leq \tau \leq 2$. Corresponding values for ChNMKh gray iron are 7% decrease ofK^* and 21% increase of c^*. It is a significant increase ofc^* in the gray iron semi-space (despite the K^* decrease) which caused a slight decrease in temperature.

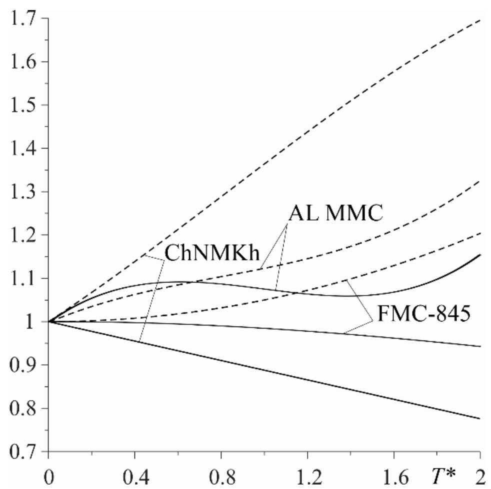

Fig. 2. Dependence of thermal conductivity K^*(solid lines) and specific heat c^* coefficient (dashed lines) on the dimensionless temperature T^* for the considered materials: ChNMKh, AL MMC, FMC-845.

In the case of the second friction couple AL MMC - FMC-845 (Fig. 3b), the difference between the temperature calculated with taking into consideration nonlinearity of materials and at constant thermo-physical properties is also increasing, but much more slower than in the case of the first friction pair. Consideration of materials thermal sensitivity causes a decrease in temperature on surfaces of both semi-spaces. For an AL MMC aluminum alloy 10% increase in coefficient of thermal conductivity K^* and 6% increase in specific heat c^* at $0 \leq \tau \leq 2$ causes, that the temperature difference calculated with and without taking into consideration the thermal sensitivity is significantly greater than cermet FMC-845.

In both considered cases, the temperature on semi-spaces surface made from cermet FMC-845 is always higher than the temperature on gray iron ChNMKh semi-spaces surface or aluminum alloy base composite AL MMC. This is due to significantly worst inferior thermal conductivity of FMC-845 when compared to ChNMKh and AL MMC (Tab. 1 and 2).

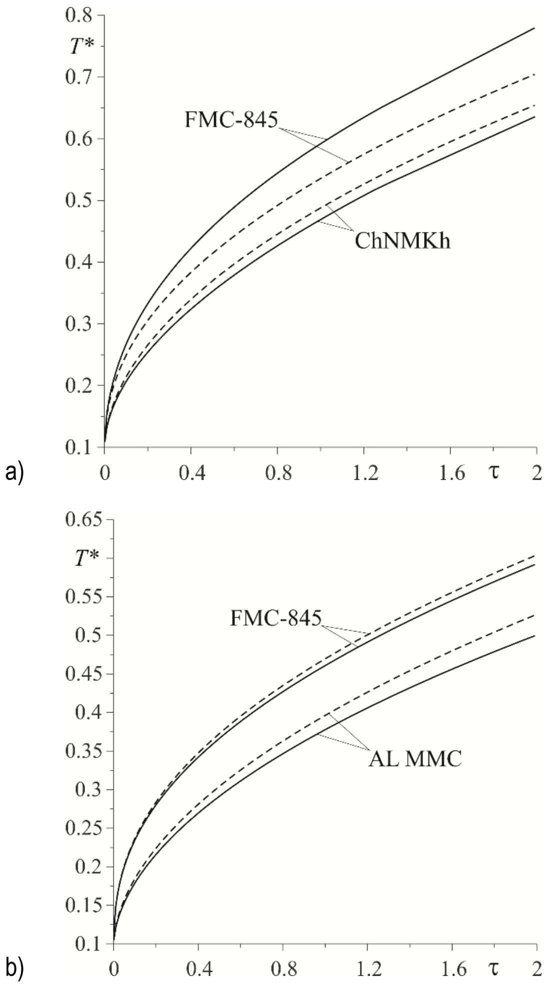

Fig. 3. Evolution of the dimensionless temperature on the contact surface for two friction pairs at $Bi = 5$: a) ChNMKh – FMC-845, b) AL MMC – FMC-845 (solid lines – calculations with considered materials thermal sensitivity; dashed lines – without materials thermal sensitivity consideration

Nomenclature: a – characteristic dimension; Bi – Biot number; c – specific heat; c_0 – specific heat at an initial temperature; $\mathrm{ierfc}(x) = \pi^{-1/2} e^{-x^2} - x\,\mathrm{erfc}(x)$, $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$, $\mathrm{erf}(x)$ – Gauss error function; f – friction coefficient; h – coefficient of thermal conductivity of contact; K – coefficient of thermal conductivity; K_0 – coefficient of thermal conductivity at an initial temperature; k – coefficient of thermal diffusivity; p_0– pressure; q_0 – specific power of friction; T – temperature; T_0 – initial temperature; T^* – dimensionless temperature; t – time; V_0 – sliding speed; z – spatial coordinate; Θ – Kirchhoff's variable; ρ – specific density; τ – Fourier number; ζ – dimensionless spatial coordinate.

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