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APPLICATION OF FUZZY RAYLEIGH DISTRIBUTION IN THE  
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Pantnagar, Uttarakhand, India, e-mail: drsurajbsingh@yahoo.co.in**Abstract**

This paper studied the implementation of fuzzy logic on the relevant parameter of biomass pyrolysis. Frequency factor, the upper limit of 'dE', and the scale parameter of Rayleigh distribution are fuzzified in order to estimate the randomness in estimating the parametric values. Distribution function,  $f(E)$ , of activation energies is assumed to follow the Rayleigh distribution. Thermo-analytical data has been found experimentally with the help of TGA/DTG analysis. The approximated solution of distributed activation energy model (DAEM) is obtained by using asymptotic approach.

**Key words:** Fuzzy number, Rayleigh distribution, distributed activation energy model, kinetic parameters, biomass pyrolysis

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**INTRODUCTION**

One of the primal tenets of modern science is that a phenomenon cannot be claimed to be well understood until it can be characterized in quantitative terms. Considering the same perspective, the constituting elements of the core scientific knowledge may be regarded as a reservoir of concepts and techniques which can be drawn upon to construct mathematical models of various types of systems and thereby yield quantitative information concerning their behaviour.

When dealing with a mathematical model, one has to pay special attention towards imprecision in data. Zadeh's (ZADEH 1978) principle of incompatibility stated that when the complexity of a system increases, our aptitude to formulate precision and meaningful statements decreases up to a threshold beyond which precision and significance became mutually exclusive characteristics. The real question is whether replacing stubbornly imprecise data by fixed ones will influence our investigation or not. If you replace arbitrarily imprecise data by the fixed values in a model, you will leave no other chance to

the model but to churn out meaningless outcomes sometime. False certainty is a wrong practice in field of science and it can be untoward if it stunts articulation of critical choices. Although the probability theory manifests the model decision making imprecision (TSOUKALAS & UHRIG 1997), yet there are various qualitative aspects of indeterminacy which are not under purview of probabilistic tools. Considering the same fact, fuzzy concept came into existence which has been implemented in many engineering problem to tackle the shortcomings of probabilistic theory.

Based on the nature of fuzzy human thinking, Lofti Zadeh propounded the “fuzzy logic” or “fuzzy set theory” in 1965. In fuzzy set theory based on fuzzy logic a particular parametric value has a degree of membership in a given set that may be anywhere in the range of 0 (completely not in the set) to 1 (completely in the set) (KERF 1975). For the same reason fuzzy logic is often defines as multi valued logic (0 to 1), compared to bi-valued Boolean logic.

In this article we propose to study fuzzification of the relevant parameters of biomass pyrolysis by using fuzzy Rayleigh distribution. The asymptotic approximation is used to estimate the kinetic parameters with the help of distributed activation energy model (DAEM). It is to be noted that the results of this paper can be used to measure randomness in the thermo-analytical data and to estimate the realistic values of parameters which in turn are used to know the kinetic mechanism of biomass.

## MATERIAL AND METHODS

### Distributed Activation Energy Model

The Distributed Activation Energy Model (DAEM) or Multiple Reaction Model (MRM) assumes that many decomposition reactions take place. It can also be comprehended as a summation of an unlimited number of parallel single step decomposition reactions, where each reaction has the following form:

$$\frac{dv_i}{dt} = A_{0i} \exp\left(-\frac{E_i}{RT}\right) (v_i^* - v_i) \quad (1)$$

where subscript  $i$  means one of several constituents,  $v_i$  is the total release mass of  $i^{\text{th}}$  constituent,  $t$  is time,  $A_{0i}$  is the frequency factor,  $E_i$  is the activation energy,  $R$  is the gas constant and  $T$  is the absolute temperature.

If the number of decomposition reactions involve is numerous, it can be assumed that activation energies of these reactions are distributed, and the reactions can be expressed as a function of the activation energy.

$$dv^* = v^* f(E) dE \quad (2)$$

The right-hand side of equation (2) expresses the fraction of maximum mass loss  $v^*$  in the given interval of activation energy. Usually,  $f(E)$  is taken to be a Gaussian distribution. However, with the selection of an appropriate distribution function for the molecular activation energies, it is advantageous to consider the Rayleigh distribution function over symmetric one, i.e. Gaussian. More, the Rayleigh is mathematically flexible.

DAEM equation for nonisothermal time dependent temperature regime can be derived by combination of equations (1) and (2) using the Rayleigh distribution and it is given as:

$$v = \int_0^\infty \left[ \exp\left(\int_{T_0}^T \frac{-A}{\theta} \exp\left(\frac{-E}{RT}\right) dT\right) \right] f(E) dE \quad (3)$$

where  $\theta$  ( $^{\circ}\text{C}/\text{min}$ ) is heating rate,  $E$  is activation energy ( $\text{kJmol}^{-1}$ ) and  $A$  is frequency factor ( $\text{s}^{-1}$ ). Constant value of the frequency factor ( $A$ ) for every decomposition reaction with various activation energies is assumed (CAI & LIU 2008). The value of the frequency factor can also be expressed as a function of activation energy or temperature.

Let  $f(E)$  is chosen to be Rayleigh Distribution with mean  $E_0$  and variance  $\sigma$  and expressed as

$$f(E) = \frac{E}{\beta^2} \exp\left(\frac{-E^2}{2\beta^2}\right) \quad (4)$$

Mean and the variance of Rayleigh distribution can be expressed as

$$E_0 = \beta \sqrt{\frac{\pi}{2}} \quad \sigma^2 = \left(\frac{4-\pi}{2}\right) \beta^2$$

### Asymptotic expansion

One of the main problems with solution of the equation (3) is the evaluation of the double integral, since it requires the large computing resources, especially when it needs to evaluate many times. In the previous approximations to the solution of equation (3), especially those seeking to convert the DAEM back into an equivalent SFOR (Single first order reaction), there is complication in extrapolating to other heating regimes and hence arise the problem in finding the volatile distribution  $f(E)$  from the thermo-analytical data. The relationship between the DAEM and the SFOR model has been explored by Niksa and Lau, which is based on approach of holding the activation energy fixed, and defining an effective or nominal rate constant  $\langle A \rangle$  (NIKSA & LAU 1993).

The characteristic of rapidly-varying double exponential function (DExp) is defined by a piecewise linear function, which has three distinct regions: DExp is zero, DExp is equal to unity, and DExp rises linearly from zero to one.

Approximation to the double exponential is, represented as

$$DExp = \exp\left(-A \int_0^t e^{-\frac{E}{RT}} dt\right) \quad (5)$$

In order to carry out the stepwise simplification of this integrand, it is very important to consider the typical values of the parameters and functions on which it depends.

The frequency factors or pre exponential term is mainly in the range of  $A \sim 10^{10} - 10^{13} s^{-1}$ , whereas the activation energies of interest are in the range 100- 300 kJ/mol. The typical values of temperatures depend on the particular experiments, but 1000-2000 K are usually used. To demonstrate the simplification method, the temperature is considered to ramp linearly with time ( $t$ ) can be expressed as  $T = \theta t$ . Then DExp becomes,

$$DExp = \exp\left(-\int_0^t A e^{-\frac{E}{R\theta u}} du\right). \quad \text{where } \theta(^{\circ}C/min) \text{ is heating rate.}$$

The integral in the exponent can be approximated by using the Laplace approach where the parameter  $\frac{E}{R\theta t}$  is assume to be large, hence the major contribution from the integral is when  $u$  is near  $t$ , or the temperature is near its maximum. This provides the well-known asymptotic approximation to DExp function:

$$DExp = \exp\left(-\int_0^t A e^{-\frac{E}{R\theta u}} du\right) \sim \exp\left(\frac{-AR\theta t^2}{E}\right) \text{ as } \frac{E}{R\theta t} \rightarrow \infty \quad (6)$$

or

Approximation of equation (5) can also be expressed in the form

$$Dexp = \exp\left(-\exp\left(\frac{E_s - E}{E_w}\right)\right),$$

where the function varies from zero to one as  $E$  increases, over a range of step size  $E_w$  around the central value  $E_s$ .

Let  $h(E) = \frac{E_s - E}{E_w}$  then equation (5) can be written as

$$Dexp = \exp\left(-\exp(h(E))\right), \quad \text{where } h(E) = \frac{-E}{R\theta t} + \ln\left(\frac{AR\theta t^2}{E}\right).$$

The behaviour of  $E_s$  is of interest, so  $h(E)$  is expanded by using Taylor series

$$h(E) \sim h(E_s) + (E - E_s)h'(E_s) + \dots$$

Now  $h(E)$ ,  $E_s$  and  $E_w$  are chosen in such a manner, that

$$h(E_s) = 0 \text{ and } h'(E_s) = \frac{-1}{E_w}.$$

After solving, we have

$$E_s = R\theta t Y(At) \text{ and } E_w = \frac{R\theta t E_s}{R\theta t + E_s}$$

where  $Y(x)$  represents Lambert W function and defined to be one of roots of the equation.

As it has been observed that DExp varies like a smooth step-function, which rises rapidly for large values of  $tA$  from zero to one in a range of activation energies of step size width  $E_w$  around the value  $E=E_s$ , where both  $E_s$  and  $E_w$  are function of time.

There are two different limits, which are applied to the distribution function  $f(E)$ . One which is relatively wide initial distribution as compared with the width of DExp, known as the wide distribution. If the initial distribution function is relatively narrow as compared with the width of DExp, defines the narrow initial distribution. The significance of the distribution type defines the shape of the total integrand, which changes with time and applied limit. When the initial distribution is relatively wide compared to  $E_w$ , the total integrand initially behaves similar to the distribution  $f(E)$ . As time proceeds, it is truncated from the left by the step- like DExp. Moreover, the location of the maximum of the total integrand varies significantly and hence the shape becomes quite skewed.

In order to apply the approach, we considered the Rayleigh distribution, centered at  $E_0$  with standard deviation  $\sigma$ .

$$\text{Approximations are found to the integral } v = \int_0^\infty \frac{E}{\beta^2} \exp(g(E)) dE,$$

where  $g(E) = -\exp\left(\frac{E_s - E}{E_w}\right) - \left(\frac{E}{\beta}\right)^2$  and  $E_s$  and  $E_w$  are functions of time ( $t$ ).

Energy is now rescaled by  $y = \frac{E}{E_0}$ . So the problem becomes

$$v = \frac{\pi}{2} \left( \int_0^\infty y \exp\left(-\exp\left(\frac{y_s - y}{y_w}\right) - \frac{\pi}{2} y^2\right) dy \right) \tag{7}$$

$$\text{and } g(y) = -\exp\left(\frac{y_s - y}{y_w}\right) - \frac{\pi}{2} y^2 \tag{8}$$

For linear ramp temperature  $T = \theta t$ ,

$$y_s = \frac{R\theta t Y(At)}{E_0}, \quad y_w = \frac{y_s}{1 + Y(At)}$$

In the wide distribution, the initial distribution is much wider than DExp. To tackle this, the limit  $\sigma y_w \ll 1$  is considered. In this limit, DExp jumps from zero to one near  $y = y_s$ , as given previously (HOWARD 1981; PITT 1962; SUUBERG 1983; VAND 1943) and been approximated by the step-function.

$$H(y - y_s) = \begin{cases} 1, & y \geq y_s \\ 0, & y < y_s \end{cases}$$

The equation (10) is rewritten in the form

$$v = \frac{\pi}{2} \int_0^\infty y \left[ \exp\left(\left(-\exp\left(\frac{y_s - y}{y_w}\right)\right)\right) - H(y - y_s) \right] \exp\left(-\frac{\pi}{2} y^2\right) dy + \frac{\pi}{2} \int_{y_s}^\infty y \exp\left(-\frac{\pi}{2} y^2\right) dy$$

$$\text{or } v = \frac{\pi}{2} \int_0^\infty y \left[ \exp\left(\left(-\exp\left(\frac{y_s - y}{y_w}\right)\right)\right) - H(y - y_s) \right] \exp\left(-\frac{\pi}{2} y^2\right) dy + (1 - C(y_s))$$

$$\text{where } C(y_s) = 1 - \exp\left(-\frac{\pi}{2} y_s^2\right)$$

Note that in the first integral, the integrand is the initial distribution multiplied by function which is very small everywhere but in a neighbourhood of size  $y_w$  around the point  $y=y_s$ . Therefore, this integrand can be approximated by expanding the initial distribution

$$D(y) \sim D(y_s) + (y - y_s)D'(y_s) + \frac{(y-y_s)^2}{2!}D''(y_s) + \frac{(y-y_s)^3}{3!}D'''(y_s) \quad \text{or}$$

$$D(y) \sim y_s \left( \exp\left(-\frac{\pi}{2}y_s^2\right) \right) \left( 1 - \frac{(y-y_s)}{y_s}(\pi y_s^2 - 1) + \pi \frac{(y-y_s)^2}{2!}(\pi y_s^2 - 3) - \frac{(y-y_s)^3}{y_s 3!} \pi ((\pi y_s^2)^2 - 6\pi y_s^2 + 3) \right)$$

Substituting  $x = \frac{y-y_s}{y_w}$ , we get

$$v = \frac{\pi}{2} \int_0^\infty y_w \left[ \exp\left(-\exp(-x)\right) - H(x) \right] y_s \left( \exp\left(-\frac{\pi}{2}y_s^2\right) \right) \left( 1 - \frac{y_w}{y_s} x (\pi y_s^2 - 1) + \pi \frac{y_w^2}{2!} x^2 (\pi y_s^2 - 3) - \frac{y_w^3 x^3}{3! y_s} \pi ((\pi y_s^2)^2 - 6\pi y_s^2 + 3) + \dots \right) dx + (1 - C(y_s))$$

Each of the integrals arising from a term in the Taylor series can be integrated separately to get the result for first order reaction ( $n=1$ ) as

$$v \sim \frac{\pi}{2} \left( \exp\left(-\frac{\pi}{2}y_s^2\right) \right) \left( y_w y_s M_0 - y_w^2 M_1 (\pi y_s^2 - 1) + \pi \frac{y_w^3 y_s}{2} M_2 (\pi y_s^2 - 3) - \frac{y_w^4}{6} \pi M_3 ((\pi y_s^2)^2 - 6\pi y_s^2 + 3) + \dots \right) + (1 - C(y_s)) \tag{9}$$

The values of coefficient  $M_n$  need only to be evaluated once, since they are independent of any parameters. The first few values are

$$M_0 \approx -0.5772, M_1 \approx -0.98906, M_2 \approx -1.81496, M_3 \approx -5.89037.$$

The values of remaining coefficients is evaluated by integral

$$M_n \equiv \int_{-\infty}^\infty x^n (e^{-e^{-x}} - H(x))$$

**Fuzzy sets**

Primarily, the fuzzy set theory was introduced by Zadeh in 1965 (ZADEH 1965), which is based on the assumption that the membership degree is equal to one minus non membership degree. In real life situation an object may or may not be member of set A to a certain degree. In other words, some hesitation about the degree of belongingness may exist. The idea of fuzzy set is in tune with human representation of reality that is more nuances than clear cut. Some philosophical related issues ranging from ontological level to application level via epistemological level may be found elsewhere (ZADEH 1978).

In a fuzzy set, the membership degree of an element is expressed by any real number from 0 to 1 rather than the limiting extremes. More formally, a fuzzy set of a set  $A \neq \phi$  is characterized by a membership function

$$\xi : A \rightarrow [0,1].$$

**Membership and non-membership functions**

Now we will consider that the activation energy, frequency factor and scale parameter ( $\beta$ ) are not clearly defined and are fuzzified, so we will replace them with trapezoidal fuzzy number as

$$\begin{aligned} \tilde{E} &= \{a_1 a_2 a_3 a_4 a_1' a_2 a_3 a_4'\} \\ \tilde{A} &= \{b_1 b_2 b_3 b_4 b_1' b_2 b_3 b_4'\} \\ \tilde{\beta} &= \{c_1 c_2 c_3 c_4 c_1' c_2 c_3 c_4'\} \end{aligned}$$

and define its membership  $\xi_{\tilde{E}}, \xi_{\tilde{A}}, \xi_{\tilde{\beta}}$  and non-membership  $\phi_{\tilde{E}}, \phi_{\tilde{A}}, \phi_{\tilde{\beta}}$  in the following manner:

$$\xi_{\tilde{E}} = \begin{cases} \frac{E - a_1}{a_2 - a_1}, & a_1 \leq E \leq a_2 \\ 1, & a_2 \leq E \leq a_3 \\ \frac{a_4 - E}{a_4 - a_3}, & a_3 \leq E \leq a_4 \\ 0, & \text{otherwise} \end{cases} \quad \xi_{\tilde{A}} = \begin{cases} \frac{A - b_1}{b_2 - b_1}, & b_1 \leq A \leq b_2 \\ 1, & b_2 \leq A \leq b_3 \\ \frac{b_4 - A}{b_4 - b_3}, & b_3 \leq A \leq b_4 \\ 0, & \text{otherwise} \end{cases}$$

$$\xi_{\tilde{\beta}} = \begin{cases} \frac{\beta - c_1}{c_2 - c_1}, & c_1 \leq \beta \leq c_2 \\ 1, & c_2 \leq \beta \leq c_3 \\ \frac{c_4 - \beta}{c_4 - c_3}, & c_3 \leq \beta \leq c_4 \\ 1, & \text{otherwise} \end{cases} \quad \phi_{\tilde{\beta}} = \begin{cases} \frac{\beta - c_1'}{c_2 - c_1'}, & c_1' \leq \beta \leq c_2 \\ 0, & c_2 \leq \beta \leq c_3 \\ \frac{c_4' - \beta}{c_4' - c_3}, & c_3 \leq \beta \leq c_4' \\ 1, & \text{otherwise} \end{cases}$$

$$\phi_{\tilde{E}} = \begin{cases} \frac{E - a_1'}{a_2 - a_1'}, & a_1' \leq E \leq a_2 \\ 0, & a_2 \leq E \leq a_3 \\ \frac{a_4' - E}{a_4' - a_3}, & a_3 \leq E \leq a_4' \\ 1, & \text{otherwise} \end{cases} \quad \phi_{\tilde{A}} = \begin{cases} \frac{A - b_1'}{b_2 - b_1'}, & b_1' \leq k_0 \leq b_2 \\ 0, & b_2 \leq k_0 \leq b_3 \\ \frac{b_4' - A}{b_4' - b_3}, & b_3 \leq k_0 \leq b_4' \\ 1, & \text{otherwise} \end{cases}$$

The  $\alpha$ -cut of above functions is obtained as follow:

$$\tilde{E}[\alpha] = \{[a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)] [a_1' + \alpha(a_2 - a_1), a_4' - \alpha(a_4' - a_3)]\}$$

$$\tilde{A}[\alpha] = \{[b_1 + \alpha(b_2 - b_1), b_4 - \alpha(b_4 - b_3)] [b_1' + \alpha(b_2 - b_1'), b_4' - \alpha(b_4' - b_3)]\}$$

$$\tilde{\beta}[\alpha] = \{[c_1 + \alpha(c_2 - c_1), c_4 - \alpha(c_4 - c_3)] [c_1' + \alpha(c_2 - c_1'), c_4' - \alpha(c_4' - c_3)]\}$$

### Fuzzy Rayleigh distribution function

If the remaining mass fraction of biomass sample is modeled by the Rayleigh distribution, then

$$f(E) = \frac{\tilde{E}}{\tilde{\beta}^2} \exp\left(\frac{-\tilde{E}^2}{2\tilde{\beta}^2}\right), E > 0, A > 0, \beta > 0,$$

where  $E$  (activation energy),  $A$  (frequency factor), and  $\beta$  (scale parameter) are crisp in nature. Let fuzzy numbers  $\tilde{E}, \tilde{A}, \tilde{\beta}$  replace  $E, A$  and  $\beta$ . Then, the fuzzy probability of obtaining a value in the interval  $[a, b]$ , is as  $\tilde{P}(m \leq X \leq n)$  and compute its  $\alpha$ -cut as follows.

$$\tilde{P}(m < v < n)[\alpha] = \left\{ \int_m^n \frac{\tilde{E}}{\tilde{\beta}^2} e^{\left[-\exp\left(\frac{E_S - \tilde{E}}{E_w}\right) - \left(\frac{\tilde{E}}{\tilde{\beta}}\right)^2\right]} dE \mid E \in \tilde{E}[\alpha] \right\} = [P^L[\alpha], P^U[\alpha]],$$

$$\tilde{P}(m < v < n)[\alpha] = \left\{ \int_m^n \frac{\tilde{E}}{\tilde{\beta}^2} e^{\left[-\exp\left(\frac{E_S - \tilde{E}}{E_w}\right) - \left(\frac{\tilde{E}}{\tilde{\beta}}\right)^2\right]} dE \mid A \in \tilde{A}[\alpha] \right\} = [P^L[\alpha], P^U[\alpha]],$$

$$\tilde{P}(m < v < n)[\alpha] = \left\{ \int_m^n \frac{\tilde{E}}{\tilde{\beta}^2} e^{\left[-\exp\left(\frac{E_S - \tilde{E}}{E_w}\right) - \left(\frac{\tilde{E}}{\tilde{\beta}}\right)^2\right]} dE \mid \beta \in \tilde{\beta}[\alpha] \right\} = [P^L[\alpha], P^U[\alpha]],$$

for all  $\alpha$ , where

$$P^L = \min \left\{ \frac{\tilde{E}}{\tilde{\beta}^2} e^{\left[-\exp\left(\frac{E_S - \tilde{E}}{E_w}\right) - \left(\frac{\tilde{E}}{\tilde{\beta}}\right)^2\right]} \mid E \in \tilde{E}[\alpha] \right\}, \quad P^U = \max \left\{ \frac{\tilde{E}}{\tilde{\beta}^2} e^{\left[-\exp\left(\frac{E_S - \tilde{E}}{E_w}\right) - \left(\frac{\tilde{E}}{\tilde{\beta}}\right)^2\right]} \mid E \in \tilde{E}[\alpha] \right\}$$

$$P^L = \min \left\{ \frac{\tilde{E}}{\tilde{\beta}^2} e^{\left[ -\exp\left(\frac{E_s - \tilde{E}}{E_w}\right) - \left(\frac{\tilde{E}}{\tilde{\beta}}\right)^2 \right]} \mid A \in \tilde{A} [\alpha] \right\}, \quad P^U = \max \left\{ \frac{\tilde{E}}{\tilde{\beta}^2} e^{\left[ -\exp\left(\frac{E_s - \tilde{E}}{E_w}\right) - \left(\frac{\tilde{E}}{\tilde{\beta}}\right)^2 \right]} \mid A \in \tilde{A} [\alpha] \right\}$$

$$P^L = \min \left\{ \frac{\tilde{E}}{\tilde{\beta}^2} e^{\left[ -\exp\left(\frac{E_s - \tilde{E}}{E_w}\right) - \left(\frac{\tilde{E}}{\tilde{\beta}}\right)^2 \right]} \mid \beta \in \tilde{\beta} [\alpha] \right\}, \quad P^U = \max \left\{ \frac{\tilde{E}}{\tilde{\beta}^2} e^{\left[ -\exp\left(\frac{E_s - \tilde{E}}{E_w}\right) - \left(\frac{\tilde{E}}{\tilde{\beta}}\right)^2 \right]} \mid \beta \in \tilde{\beta} [\alpha] \right\}$$

## APPLICATION

For applications point of view, the experiment has been conducted for the non-isothermal pyrolysis of pine needles. It is to be noted that the result of this paper has been used in the fuzzification process to obtain the randomness and authentic values of parameters. Table 1 indicates the chemical composition which is also obtained with the help of CHNO-S analysis of pine needle samples.

**Table 1:** Chemical composition of pine needles

Biomass Type	C	H	N	O	V.M*	H.H.V**	S	Ash
Pine Needle	53.64	5.36	0.62	33.92	68.4	20.8	0.20	2.1

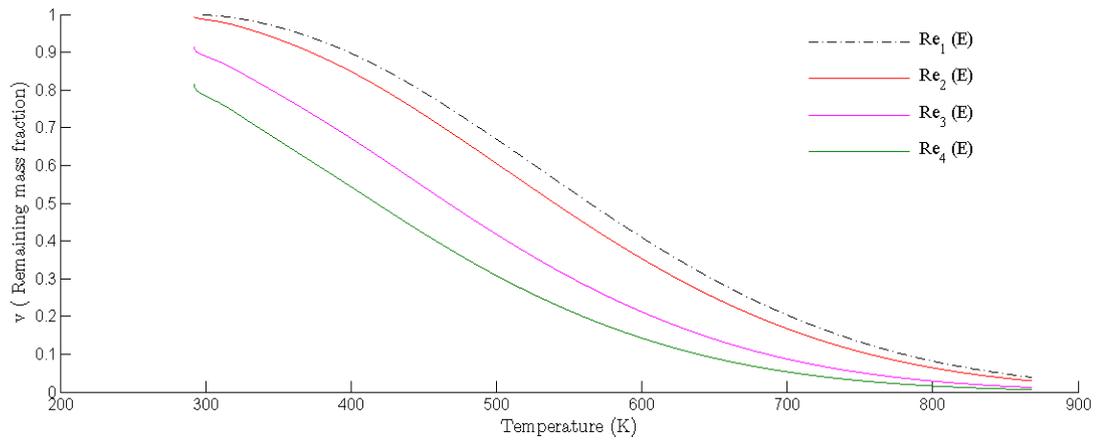
\*Volatile matter \*\*High heating value

## RESULTS AND DISCUSSION

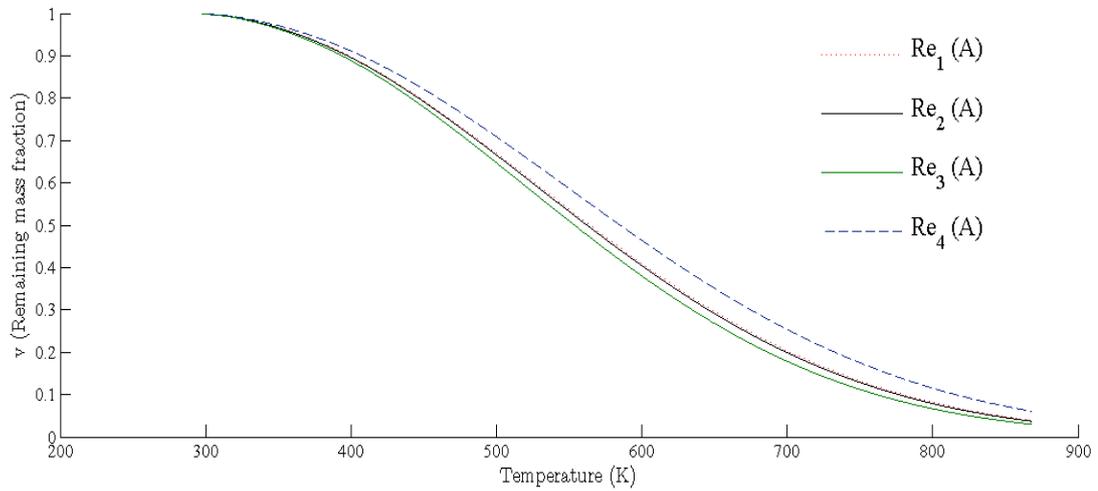
### Numerical illustration

Supposing the initial distribution of activation energy follows the Rayleigh distribution with fuzzy scale parameters  $\tilde{E}$ ,  $\tilde{A}$  and  $\tilde{\beta}$  taken as the trapezoidal fuzzy number. The values of parameters obtained after fuzzification are taken at  $\alpha=0$ .

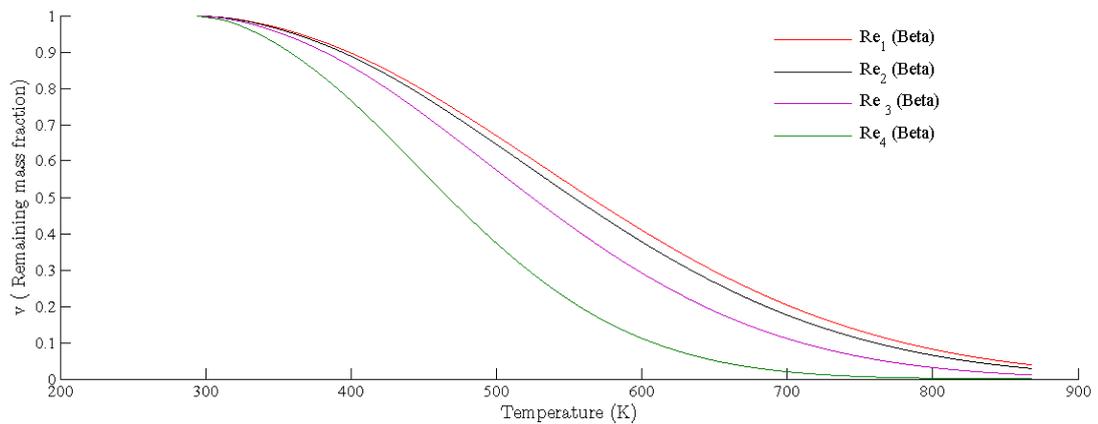
Let  $\tilde{E} (kJmol^{-1}) = [(119.66, 129.6, 149.6, 159.66)]$ ,  $\tilde{A} (s^{-1}) = [(15e + 11, 18e + 11, 42e + 11, 24e + 10)]$  and  $\tilde{\beta} (kJmol^{-1}) = [(47, 45, 40, 30)]$ . The effect of fuzzified parameters with temperature is illustrated by means of fuzzified bands  $Re_1, Re_2, Re_3, Re_4$ . The  $Re_1$  and  $Re_2$  represent the fuzzified bands corresponding to membership function; whereas  $Re_3$  and  $Re_4$  represent those corresponding to non-membership function. After the fuzzy analysis of outer limit of “dE” integral,  $\tilde{E}$ , we have evaluated four fuzzified bands which are depicted in Figure 1. If the nature of the outer limit of “dE” integral,  $E$  had been crisp, all the four fuzzified bands would have been converged to a single band. In the beginning of pyrolysis process, the remaining mass proportion must be close to one. From the fuzzified bands obtained in Figure 1, it has been observed that the remaining mass fraction curves is less than one for fuzzy set interval (149.6, 159.66), whereas the remaining mass fraction agrees to thermo analytical data for the narrow fuzzy set interval (119.66, 129.6). The effect of the fuzzified frequency factor on the numerical results is shown in Figure 2. According to the fuzzified bands, increase in fuzzy interval causes the remaining mass fraction curves to shift up. However, the membership function for an interval (15e + 11, 18e + 11) provides the good fit with experimental data. Influence of scale parameter of fuzzy Rayleigh distribution on the fuzzy analysis is depicted in Figure 3, where it is observed that the inflexion point of curves approach to zero, and the mass fraction curves is shift down the temperature scale by increase in the fuzzy band size of non-membership function.



**Fig. 1:** The effect of fuzzified upper limit of “dE” integral on the numerical solution of DAEM



**Fig. 2:** The effect of fuzzified frequency factor (A) (s-1) on the numerical solution of DAEM



**Fig. 3:** The effect of fuzzified scale parameter ( $\beta$ ) ( $\text{kJmol}^{-1}$ ) of Rayleigh Distribution on the numerical solution of DAEM

## CONCLUSION

In this article the kinetic parameters of DAEM have considered in fuzzy form. The fuzzy Rayleigh distribution has been successfully applied to the DAEM. Whenever, the distribution of activation energies and the parameters of distribution function contain randomness and fuzziness respectively, the conventional system is found to be infeasible. Thus, in order to overcome this complication, we have successfully implemented the fuzzy logic. The use of fuzzified kinetic parameters in the DAEM has overshadowed the demerits of the crisp-kinetic parameters, as the crisp data doesn't provide the membership interval for which the model can be approximated to some other realistic values. The fuzzy sets have handled the randomness or fuzziness to a certain desired level of accuracy, which in turn helped in making our analysis more precise and authentic.

The membership and the non-membership function of fuzzy set have been computed. Using this method, the fuzzified relevant parameters of biomass pyrolysis have been evaluated. It has been observed that the membership function provides good simulation with thermo analytical data for relatively small interval rather than large size of fuzzy set.

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